Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Monday 22 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Solve the equation

$$2\cosh^2 x - 3\sinh x = 1,$$

giving your answers in terms of natural logarithms.

(6)

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2. A curve has equation

$$y = \cosh x, \qquad 1 \leq x \leq \ln 5.$$

Find the exact length of this curve. Give your answer in terms of e.

(5)

3.
$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

(*a*) Find the eigenvalues of **A**.

(5)

(*b*) Find a normalised eigenvector for each of the eigenvalues of **A**.

(5)

(*c*) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{D}$.

(2)

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4. The curve C has equation

$$y = \frac{1}{\sqrt{x^2 + 2x - 3}}, \qquad x > 1.$$

(*a*) Find $\int y \, \mathrm{d}x$.

(3)

The region R is bounded by the curve C, the x-axis and the lines with equations $x = 2$ and $x = 3$. The region R is rotated through 2π radians about the x-axis.

(*b*) Find the volume of the solid generated. Give your answer in the form $p\pi \ln q$, where p and q are rational numbers to be found.

(4)

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5. The points A, B and C have position vectors $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ respectively.

(*a*) Find a vector equation of the straight line AB.

(2)

(*b*) Find a cartesian form of the equation of the straight line AB.

(2)

The plane Π contains the points A, B and C.

(*c*) Find a vector equation of Π in the form $\mathbf{r}.\mathbf{n} = p$.

(4)

(*d*) Find the perpendicular distance from the origin to Π.

(2)

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6. The hyperbola H is given by the equation $x^2 - y^2 = 1$

(*a*) Write down the equations of the two asymptotes of H.

(1)

(*b*) Show that an equation of the tangent to H at the point P $(\cosh t, \sinh t)$ is

$$y \sinh t = x \cosh t - 1.$$

(3)

The tangent at P meets the asymptotes of H at the points Q and R.

(*c*) Show that P is the midpoint of QR.

(3)

(*d*) Show that the area of the triangle OQR, where O is the origin, is independent of t.

(3)

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7. $$I_n = \int \sin^n x \, dx \quad , \quad n \geq 0.$$

(*a*) Prove that for $n \geq 2$,

$$In = \frac{1}{n}(-\sin^{n-1} x \cos x + (n-1)I_{n-2}).$$

(4)

Given that n is an odd number, $n \geq 3$,

(*b*) show that

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx = \frac{(n-1)(n-3)...6.4.2}{n(n-2)(n-4)...7.5.3}.$$

(4)

(*c*) Hence find $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x \, dx$.

(3)

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8. The ellipse E has equation $x^2 + 4y^2 = 4$

(*a*) (i) Find the coordinates of the foci, F_1 and F_2, of E.

(ii) Write down the equations of the directrices of E.

(4)

(*b*) Given that the point P lies on the ellipse, show that

$$|PF_1| + |PF_2| = 4.$$

(4)

A chord of an ellipse is a line segment joining two points on the ellipse.

The set of midpoints of the parallel chords of E with gradient m, where m is a constant, lie on a straight line l.

(*c*) Find an equation of l.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Notes	Marks
1.	$2\left(1+\sinh^2 x\right)-3\sinh x=1$	Attempt to use $\cosh^2 x=1+\sinh^2 x$	M1
	$2\sinh^2 x-3\sinh x+1=0$	Correct 3 term quadratic. The "= 0" may be implied by their attempt to solve.	A1
	$(2\sinh x-1)(\sinh x-1)=0$	Attempts to solve their 3TQ = 0 leading to $\sinh x=$ ... (= 0 may be implied)	M1
	$\sinh x$ or $\frac{e^x-e^{-x}}{2}=\frac{1}{2}$ or 1	Both values correct	A1
	$x=\ln\frac{1}{2}\left(1+\sqrt{5}\right),\ \ln\left(1+\sqrt{2}\right)$	A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ or $\ln\left(1+\sqrt{2}\right)$ oe A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ and $\ln\left(1+\sqrt{2}\right)$ oe and **no other values**	A1, A1 **M1A1 on ePEN**
	Allow equivalent answers e.g. $\ln\left(\frac{1}{2}+\sqrt{\frac{5}{4}}\right),\ln\left(\frac{1}{2}+\sqrt{1+\frac{1}{4}}\right)$ and allow awrt 3SF accuracy e.g. $\ln 1.62$, $\ln 2.41$		
			(6)
			Total 6
	Alternative		
	$2\left(\frac{e^x+e^{-x}}{2}\right)^2-3\left(\frac{e^x-e^{-x}}{2}\right)=1$	Substitutes **correct** definitions for sinh*x* and cosh*x* in terms of exponentials	M1
	$e^{4x}-3e^{3x}+3e^x+1=0$	Correct quartic in e^x	A1
	$\left(e^{2x}-e^x-1\right)\left(e^{2x}-2e^x-1\right)=0\Rightarrow e^x=...$	Solves their quartic as far as e^x = ... For the **correct quartic** there must be a recognisable attempt to solve e.g. the product of two 3TQ's in e^x or if answers only are given, they must be correct (1.62, 2.41, and possibly (-0.618, -0.414)). For an **incorrect quartic** there must be a recognisable attempt to solve a quartic with at least 4 terms.	M1
	$e^x=\frac{1+\sqrt{5}}{2},\ \frac{2+\sqrt{8}}{2}$	Correct values for e^x. Allow $e^x=\frac{1\pm\sqrt{5}}{2},\ \frac{2\pm\sqrt{8}}{2}$ **but no incorrect values**. Allow awrt 1.62, 2.41	A1
	$x=\ln\frac{1}{2}\left(1+\sqrt{5}\right),\ \ln\left(1+\sqrt{2}\right)$	A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ or $\ln\left(1+\sqrt{2}\right)$ oe A1: $x=\ln\frac{1}{2}\left(1+\sqrt{5}\right)$ and $\ln\left(1+\sqrt{2}\right)$ oe and **no other values**. allow awrt 3SF accuracy e.g. $\ln 1.62$, $\ln 2.41$	A1, A1 **M1A1 on ePEN**

Question Number	Scheme	Notes	Marks
2	$y=\cosh x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\sinh x$	Correct derivative	B1
	$\int\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\,\mathrm{d}x=\int\sqrt{1+\sinh^2 x}\,\mathrm{d}x$	Uses the correct formula with their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	Alternative for first 2 marks: $y=\frac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x-\mathrm{e}^{-x}}{2}=$ B1 $\int\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\,\mathrm{d}x=\int\sqrt{1+\left(\frac{\mathrm{e}^x-\mathrm{e}^{-x}}{2}\right)^2}\,\mathrm{d}x=$ M1 Then apply the scheme		
	$=\int\cosh x\,\mathrm{d}x$ or $\int\frac{e^x+e^{-x}}{2}\mathrm{d}x$	Correct integral (Condone omission of dx)	A1
	$=[\sinh x]_1^{\ln 5}=\sinh(\ln 5)-\sinh(1)$	$\int\cosh x\,\mathrm{d}x=\sinh x$ and correct use of the correct limits. **Dependent on the first method mark.**	dM1
	$=\frac{12}{5}-\frac{1}{2}\left(\mathrm{e}-\frac{1}{\mathrm{e}}\right)$	Or equivalent (must be in terms of e with no ln's) **Score when a correct answer is first seen and isw.**	A1cso
			(5)
	Some equivalent final answers: $\frac{12}{5}-\frac{\mathrm{e}}{2}+\frac{\mathrm{e}^{-1}}{2}$, $2.4-\frac{\mathrm{e}-\mathrm{e}^{-1}}{2}$, $\frac{12}{5}-\frac{\mathrm{e}^2-1}{2\mathrm{e}}$, $\frac{24\mathrm{e}-5\mathrm{e}^2+5}{10\mathrm{e}}$		
	Special Case: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\sinh x$ le**ads to a correct answer. This scores a maximum of 3/5 i.e. B0M1A1(recovery)dM1A0**		
			Total 5

Question Number	Scheme	Notes	Marks
3(a)	$\det(\mathbf{A}-\lambda\mathbf{I})=0$ or $\begin{vmatrix}2-\lambda & 1 & 0\\ 1 & 2-\lambda & 1\\ 0 & 1 & 2-\lambda\end{vmatrix}=0$	Either statement is sufficient. May also be implied by an attempt to form the characteristic equation	M1
	$(2-\lambda)((2-\lambda)^2-1)-(2-\lambda)=0$ or $(2-\lambda)\left[(2-\lambda)^2-2\right]=0$ $\left(\lambda^3-6\lambda^2+10\lambda-4=0\right)$	Recognisable attempt at characteristic equation – sign errors only.	M1
	$(2-\lambda)((\lambda^2-4\lambda+2))=0$		
	$\lambda=2,\ \ 2+\sqrt{2},\ \ 2-\sqrt{2}$ Allow awrt 3.41 and 0.586	B1: $\lambda=2$ **from any working** M1: Attempt to solve (usual rules) $\lambda^2-4\lambda+2=0$ A1: Obtains $2\pm\sqrt{2}$ oe e.g. $\frac{4\pm\sqrt{8}}{2}$	B1M1A1
			(5)
(b)	$\begin{pmatrix}2&1&0\\1&2&1\\0&1&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=2\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $(2+\sqrt{2})\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $(2-\sqrt{2})\begin{pmatrix}x\\y\\z\end{pmatrix}$ States or uses $\mathbf{A}\boldsymbol{x}=\lambda\boldsymbol{x}$ or $(\mathbf{A}-\lambda\mathbf{I})\boldsymbol{x}=\mathbf{0}$ for at least one of their eigenvalues		M1
	$\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}1\\\sqrt{2}\\1\end{pmatrix},\begin{pmatrix}1\\-\sqrt{2}\\1\end{pmatrix}$ (any multiple of these)	A1: One correct eigenvector (allow awrt 1.41 for $\sqrt{2}$) A1: Two correct eigenvectors (allow awrt 1.41 for $\sqrt{2}$) A1: All eigenvectors correct (allow awrt 1.41 for $\sqrt{2}$)	A1 A1 A1 **No ft here**
	$\pm\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\-\frac{1}{\sqrt{2}}\end{pmatrix},\pm\begin{pmatrix}\frac{1}{2}\\\frac{1}{\sqrt{2}}\\\frac{1}{2}\end{pmatrix},\pm\begin{pmatrix}\frac{1}{2}\\-\frac{1}{\sqrt{2}}\\\frac{1}{2}\end{pmatrix}$	All normalised and correct and exact. Allow equivalent forms e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$ (**Must be seen in (b))**	A1 **No ft here**
			(5)
(c)	$\mathbf{P}=\begin{pmatrix}\frac{1}{\sqrt{2}}&\frac{1}{2}&\frac{1}{2}\\0&\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}&\frac{1}{2}&\frac{1}{2}\end{pmatrix}$ **D=** $\begin{pmatrix}2&0&0\\0&2+\sqrt{2}&0\\0&0&2-\sqrt{2}\end{pmatrix}$	B1ft: One correct ft matrix. If awarding for **P** they must be using their normalised vectors B1ft: Both correct ft matrices and **P** consistent with **D**. The eigenvectors in **P** must be in the same order as the eigenvalues in **D**. **For both B marks it must be clear or implied which matrix is which. (NB: B0B1 is not possible)**	B1ft, B1ft
			(2)
			Total 12

Question Number	Scheme	Notes	Marks
4(a)	$x^2+2x-3=(x+1)^2-4$	$x^2+2x-3=(x\pm1)^2\pm\alpha\pm3,\ \alpha\neq0$	M1
	$\int\frac{1}{\sqrt{(x+1)^2-4}}\,dx=\text{arcosh}\ \frac{(x+1)}{2}(+c)$ or $\ln\left\{(x+1)^2+\sqrt{(x+1)^2-4}\right\}$	M1: Use of arcosh (allow arccosh, $\cosh^{-1}$) Or uses $\ln\left\{x+\sqrt{x^2-a^2}\right\}$ A1: arcosh $\frac{(x+1)}{2}$ (+ c not required) Or $\ln\left\{(x+1)+\sqrt{(x+1)^2-4}\right\}$	M1 A1
			(3)
(b)	$S=\pi\int y^2dx=\pi\int\left(\frac{1}{\sqrt{x^2+2x-3}}\right)^2dx$	Use of $\int\pi y^2dx$	M1
	$=\int\frac{1}{(x+1)^2-4}\,dx=\left[\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)\right]$	M1: Use of $\ln\left(\frac{x\pm p}{x\pm q}\right)$ A1: $\int\frac{1}{(x+1)^2-4}\,dx=\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)$	M1A1
	$= =\frac{\pi}{4}\left(\ln\frac{1}{3}-\ln\frac{1}{5}\right)=\frac{\pi}{4}\ln\frac{5}{3}$	$\frac{\pi}{4}\ln\frac{5}{3}$	A1
	Special case: Uses $S=k\int y^2dx$ scores a maximum M0M1A1A0		
			(4)
	NB: May use partial fractions in (b) for middle M1A1:		
	$\frac{1}{x^2+2x-3}\equiv\frac{1}{(x+3)(x-1)}\equiv\frac{1}{4}\left(\frac{1}{x-1}-\frac{1}{x+3}\right)$		
	$\int\frac{1}{(x+3)(x-1)}\,dx=\left[\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)\right]$	M1: Use of $\ln\left(\frac{x\pm p}{x\pm q}\right)$ A1: $\frac{1}{4}\ln\left(\frac{x-1}{x+3}\right)$	M1A1
	Alternative for (b) by substitution:		
	$S=\pi\int y^2dx=\pi\int\left(\frac{1}{\sqrt{x^2+2x-3}}\right)^2dx$	Use of $\int\pi y^2dx$	M1
	$u=x+1\Rightarrow\int\frac{1}{(x+1)^2-4}\,dx=\int\frac{1}{u^2-4}\,du$		
	$\int\frac{1}{u^2-4}\,du=\left[\frac{1}{4}\ln\frac{u-2}{u+2}\right]$	M1: Use of $\ln\left(\frac{u\pm p}{u\pm q}\right)$ A1: $\frac{1}{4}\ln\frac{u-2}{u+2}$	M1A1
	$\pi\left[\frac{1}{4}\ln\frac{u-2}{u+2}\right]_3^4=\frac{\pi}{4}\left(\ln\frac{1}{3}-\ln\frac{1}{5}\right)=\frac{\pi}{4}\ln\frac{5}{3}$	$\frac{\pi}{4}\ln\frac{5}{3}$	A1
			Total 7

Question Number	Scheme	Notes	Marks
5(a)	$\mathbf{AB} = -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	Attempt $\pm(\mathbf{OB} - \mathbf{OA})$	M1
	$\mathbf{r} = \begin{pmatrix}1\\3\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\left(\mathbf{r} - \begin{pmatrix}1\\3\\2\end{pmatrix}\right) \times \begin{pmatrix}-2\\-3\\-1\end{pmatrix} = 0$	Any correct vector form including the "**r** = " and the "= 0" "**r** = " can be "AB =" or "*l* = "etc. The direction can be any multiple of that shown.	A1
			(2)
(b)	$\dfrac{x-"1"}{"-2"} = \dfrac{y-"3"}{"-3"} = \dfrac{z-"2"}{"-1"}$ oe e.g. $\dfrac{x+1}{2} = \dfrac{y}{3} = \dfrac{z-1}{1}$	M1: Correct attempt at the Cartesian form using their position and direction A1: $\dfrac{x-1}{-2} = \dfrac{y-3}{-3} = \dfrac{z-2}{-1}$ oe	M1A1
			(2)
(c)	$\mathbf{AB} \times \mathbf{AC} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & -3 & -1\\ 1 & -2 & -2\end{vmatrix} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix} \times \begin{pmatrix}1\\-2\\-2\end{pmatrix} = \begin{pmatrix}4\\-5\\7\end{pmatrix}$ $(= \mathbf{AB} \times \mathbf{BC} = \mathbf{AC} \times \mathbf{BC})$	M1: Attempts vector product of 2 vectors **in the plane** e.g. $\mathbf{AB} \times \mathbf{BC}$ If there is no working, at least 2 components should be correct. A1: Any multiple of $4\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$	M1A1
	$\mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = \begin{pmatrix}1\\3\\2\end{pmatrix}.\begin{pmatrix}4\\-5\\7\end{pmatrix}$ i.e. $\mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = 3$ oe e.g. $\mathbf{r}.\begin{pmatrix}-4\\5\\-7\end{pmatrix} = -3$	dM1: Attempts scalar product using their normal vector and **a**, **b** or **c**. **Dependent on the previous M** A1: Correct equation (oe)	dM1A1
	See end of scheme for alternatives		
			(4)
(d)	$d = \dfrac{"3"}{\lvert "\mathbf{4i} - 5\mathbf{j} + \mathbf{7k}" \rvert} = \dfrac{3}{\sqrt{90}}$	M1: $d = \dfrac{\pm \text{their } p}{\lvert \text{their } \mathbf{n} \rvert}, \quad p \neq 0$ A1: $\dfrac{3}{\sqrt{90}}$ oe e.g. $\dfrac{3}{3\sqrt{10}}, \dfrac{1}{\sqrt{10}}$, (awrt 0.316)	M1A1 **Note B1B1 on ePEN**
			(2)
	Alternative		
	$\lambda\begin{pmatrix}4\\-5\\7\end{pmatrix}.\begin{pmatrix}4\\-5\\7\end{pmatrix} = 3 \Rightarrow \lambda = \dfrac{1}{30} \Rightarrow d = \sqrt{\left(\dfrac{4}{30}\right)^2 + \left(\dfrac{5}{30}\right)^2 + \left(\dfrac{7}{30}\right)^2} = \dfrac{1}{\sqrt{10}}$		M1A1 **Note B1B1 on ePEN**
	M1: A correct method for finding "λ" and attempting the length of λ**n** A1: $\dfrac{3}{\sqrt{90}}$ oe e.g. $\dfrac{3}{3\sqrt{10}}, \dfrac{1}{\sqrt{10}}$, (awrt 0.316)		

Question Number	Scheme	Notes	Marks
6(a)	$y = x, y = -x$	Both required. Accept $y = \pm x$ and $x = \pm y$	B1
			(1)
(b)	$\frac{dy}{dx} = \frac{\cosh t}{\sinh t}$	Correct gradient	B1 **Note M1 on ePEN**
	$y - \sinh t = \frac{\cosh t}{\sinh t}(x - \cosh t)$	Correct straight line method. For $y = mx + c$ method, c must be found	M1
	$y \sinh t = x \cosh t - (\cosh^2 t - \sinh^2 t)$		
	$y \sinh t = x \cosh t - 1$*	Obtains the printed answer with at least one intermediate step.	A1* cso
			(3)
(c)	$y = x \Rightarrow x = \frac{1}{\cosh t - \sinh t}, y = \frac{1}{\cosh t - \sinh t}$ $y = -x \Rightarrow x = \frac{1}{\cosh t + \sinh t}, y = \frac{-1}{\cosh t + \sinh t}$	All four values correct. May be in exponential form e.g. (e^t, e^t) and $(e^{-t}, -e^{-t})$	B1
	$X = \frac{1}{2}\left(\frac{1}{\cosh t - \sinh t} + \frac{1}{\cosh t + \sinh t}\right)$ or $Y = \frac{1}{2}\left(\frac{1}{\cosh t - \sinh t} + \frac{-1}{\cosh t + \sinh t}\right)$	Correct attempt at X or Y. May be in exponential form e.g. $\left(\frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2}\right)$	M1
	$X = \frac{1}{2}\left(\frac{\cosh t + \sinh t + \cosh t - \sinh t}{\cosh^2 t - \sinh^2 t}\right) = \cosh t$ $Y = \frac{1}{2}\left(\frac{\cosh t + \sinh t - \cosh t + \sinh t}{\cosh^2 t - \sinh^2 t}\right) = \sinh t$	Obtains $X = \cosh t$ and $Y = \sinh t$ May be shown using exponentials as above.	A1cso
			(3)
(d)	$A = \frac{1}{2}\sqrt{\frac{2}{(\cosh t - \sinh t)^2}} \cdot \sqrt{\frac{2}{(\cosh t + \sinh t)^2}}$ Or e.g. $\frac{1}{2}\sqrt{2e^{2t}}\sqrt{2e^{-2t}}$	Correct triangle area method	M1
	$= \frac{1}{\cosh^2 t - \sinh^2 t} = 1$	Obtains an area of 1	A1
	So area is independent of t	Concludes independence of t having obtained a constant area. Conclusion must include the word independent (or not dependent) (but not e.g. just QED)	A1ft
			(3)
	Alternative area method: If A$\left(\frac{1}{\cosh t}, 0\right)$ is the intersection of QR with the x-axis Area OAR + Area OAQ = $\frac{1}{2} \times \frac{1}{\cosh t} \times \frac{1}{\cosh t - \sinh t} + \frac{1}{2} \times \frac{1}{\cosh t} \times \frac{1}{\cosh t + \sinh t}$ $= \frac{1}{2} \times \frac{1}{\cosh t} \times \left(\frac{1}{\cosh t - \sinh t} + \frac{1}{\cosh t + \sinh t}\right) = \frac{1}{2\cosh t} \times 2\cosh t = 1$		
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$I_n = \int \sin^{n-1} x \sin x \mathrm{d}x$	Split into $\sin^{n-1} x$ and $\sin x$	M1
	$I_n = \sin^{n-1} x(-\cos x) + \int (n-1)\sin^{n-2} x \cos^2 x \mathrm{d}x$	Integration by parts in the right direction (if the method is unclear or formula not quoted only allow sign errors) **Dependent on the first method mark.**	dM1
	$I_n = -\sin^{n-1} x \cos x + (n-1)(I_{n-2} - I_n)$	Obtains I_n correctly in terms of I_{n-2} and I_n	A1
	$I_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2} - nI_n + I_n$		
	$I_n = \frac{1}{n}(-\sin^{n-1} x \cos x + (n-1)I_{n-2})$ *	**Printed answer** obtained with at least one intermediate step and no errors seen (condone the occasional x lost along the way but the final answer must be **exactly** as printed)	A1*
	Condone omission of "dx" throughout in both methods		**(4)**
	Alternative:		
	$= \int \sin^{n-2} x\left(1-\cos^2 x\right)\mathrm{d}x$	Splits into $\sin^{n-2} x$ and $\sin^2 x$ and uses $\sin^2 x = 1 - \cos^2 x$	M1
	$= I_{n-2} - \left\{\frac{\sin^{n-1} x \cos x}{n-1} + \int \frac{\sin^n x}{n-1}\mathrm{d}x\right\}$	Integration by parts in the right direction (if the method is unclear or formula not quoted only allow sign errors). **Dependent on the first method mark.**	dM1
	$= I_{n-2} - \frac{\sin^{n-1} x \cos x}{n-1} - \frac{1}{n-1} I_n$	Obtains I_n correctly in terms of I_{n-2} and I_n	A1
	$\left(n-1\right)I_n = \left(n-1\right)I_{n-2} - \sin^{n-1} x \cos x - I_n$		
	$I_n = \frac{1}{n}(-\sin^{n-1} x \cos x + (n-1)I_{n-2})$ *	Printed answer obtained with at least one intermediate step and no errors seen ((condone the occasional x lost along the way but the final answer must be **exactly** as printed)	A1*
(b)	$I_n = \frac{1}{n}(\left[-\sin^{n-1} x \cos x\right]_0^{\frac{\pi}{2}} + (n-1)I_{n-2})$	Use part (a) with limits	M1
	$I_n = \frac{n-1}{n} I_{n-2}$	Sight of the expression could score M1A1	A1
	n odd, $I_1 = \int_0^{\frac{\pi}{2}} \sin x \mathrm{d}x = \left[-\cos x\right]_0^{\frac{\pi}{2}} = 1$	An attempt at I_1 must be seen before any more marks are awarded	
	$I_n = \frac{(n-1)}{n} I_{n-2} = \frac{(n-1)}{n}\frac{(n-3)}{n-2} I_{n-4} = ...$	Attempts I_1 and at least 2 fractions in terms of n	M1
	$I_n = \frac{(n-1)(n-3)...6.4.2}{n(n-2)(n-4)...7.5.3}$ **	Cso. Note this may be awarded for 'extra' brackets top and bottom provided all previous marks are scored.	A1**
			(4)
(c)	$\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^5 x(1-\sin^2 x)\ \mathrm{d}x$	Uses $\cos^2 x = 1 - \sin^2 x$	M1
	$= I_5 - I_7 = \frac{4\times 2}{5\times 3} - \frac{6\times 4\times 2}{7\times 5\times 3}$	Correct numerical expression	A1
	$= \frac{8}{105}$	Cao (accept awrt 0.0761)	A1
	Correct answer only with no working would generally score no marks		**(3)**
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2(1-e^2) \Rightarrow e^2 = \frac{3}{4}$ or $e = \frac{\sqrt{3}}{2}$ NB $a = 2$, $b = 1$	M1: Uses a **correct** eccentricity formula to find a value for e or e^2 A1: $e^2 = \frac{3}{4}$ or $e = \frac{\sqrt{3}}{2}$ (allow $e = \pm\frac{\sqrt{3}}{2}$)	M1A1
	Foci: $(\pm ae, 0) \Rightarrow (\pm\sqrt{3}, 0)$	Both correct as coordinates	B1
	Directrices: $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{4}{\sqrt{3}}$	Both directrices correct seen as equations. Accept un-simplified e.g. $x = \pm\frac{2}{\sqrt{3}/2}$	B1
			(4)
(b)	$PF_1 = ePN_1$ and $PF_2 = ePN_2$	Use of definition of ellipse for either PF_1 or PF_2	M1
	$PF_1 + PF_2 = e(PN_1 + PN_2) = eN_1N_2$	dM1: $(\text{their } e) \times 2\left(\text{their } \frac{4}{\sqrt{3}}\right)$ **Dependent on the previous method mark** A1: $\frac{\sqrt{3}}{2} \times \left(2 \times \frac{4}{\sqrt{3}}\right)$	dM1A1
	$= 4$**	cso	A1**
			(4)
	(b) Alternative 1: Using $P\left(2\cos\theta, \sin\theta\right)$ **(Must be of this form)**		
	$PF_1 = \sqrt{(2\cos\theta - \sqrt{3})^2 + \sin^2\theta}$ $PF_2 = \sqrt{(2\cos\theta + \sqrt{3})^2 + \sin^2\theta}$	Correct use of Pythagoras for either PF_1 or PF_2	M1
	$PF_1 = \sqrt{(2 - \sqrt{3}\cos\theta)^2}$ and $PF_2 = \sqrt{(2 + \sqrt{3}\cos\theta)^2}$ dM1: Obtains both $PF_1{}^2 = \left(\sqrt{3}\cos\theta - \sqrt{p^2+1}\right)^2$ and $PF_2{}^2 = \left(\sqrt{3}\cos\theta + \sqrt{p^2+1}\right)^2$ where p is the x-coordinate of a focus. **Dependent on the previous method mark**		dM1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = 2 - \sqrt{3}\cos\theta + 2 + \sqrt{3}\cos\theta$	$2 - \sqrt{3}\cos\theta + 2 + \sqrt{3}\cos\theta$. Note that if $\sqrt{3}\cos\theta - 2$ is obtained correctly, it must become $2 - \sqrt{3}\cos\theta$ to score any A marks	A1
	$= 4$**	cso	A1**
	(b) Alternative 2: Using $P\left(x, \sqrt{\frac{4-x^2}{4}}\right)$ **(Must be of this form)** or $P\left(\sqrt{4-4y^2}, y\right)$		
	$PF_1 = \sqrt{(x-\sqrt{3})^2 + \frac{4-x^2}{4}}$ $PF_2 = \sqrt{(x+\sqrt{3})^2 + \frac{4-x^2}{4}}$	Correct use of Pythagoras for either PF_1 or PF_2	M1
	$PF_1 = \sqrt{\left(2 + \frac{\sqrt{3}}{2}x\right)^2}$ and $PF_2 = \sqrt{\left(2 - \frac{\sqrt{3}}{2}x\right)^2}$ dM1: Obtains both $PF_1{}^2 = \left(\frac{\sqrt{3}}{2}x - \sqrt{p^2+1}\right)^2$ and $PF_2{}^2 = \left(\frac{\sqrt{3}}{2}x + \sqrt{p^2+1}\right)^2$ where p is the x-coordinate of the foci. **Dependent on the previous method mark**		dM1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = 2 - \frac{\sqrt{3}}{2}x + 2 + \frac{\sqrt{3}}{2}x$	$2 - \frac{\sqrt{3}}{2}x + 2 + \frac{\sqrt{3}}{2}x$	A1
	$= 4$**	cso	A1**

(c)	Using chord as $y = mx + c$		
	$\frac{x^2}{4} + (mx+c)^2 = 1$	Substitutes the equation of a straight line with gradient *m* into the equation of the ellipse	M1
	$(1+4m^2)x^2 + 8mcx + 4(c^2-1) = 0$	Correct quadratic in *x* with terms collected	A1
	$x = \frac{1}{2}(\text{sum of roots}) = \frac{-4mc}{1+4m^2}$	Attempts $\frac{1}{2}(\text{sum of roots})$	M1
	$\Rightarrow c = -\frac{(1+4m^2)x}{4m}$	Correct expression for *c* in terms of *m* and *x*	A1
	So $y = mx - \frac{(1+4m^2)x}{4m}\left(= -\frac{1}{4m}x\right)$	ddM1: Substitutes back into $y = mx + c$ **Depends on both previous method marks** A1: Correct equation	ddM1A1
			(6)
	Or for last 3 marks:		
	$x = \frac{-4mc}{1+4m^2} \Rightarrow y = \frac{-4m^2c}{1+4m^2} + c\left(= \frac{c}{1+4m^2}\right)$	Correct *y*-coordinate in terms of *m* and *c*.	A1
	$y = -\frac{1}{4m}x$	ddM1: Obtains *y* in terms of *x* and *m* **Depends on both previous method marks** A1: Correct equation	ddM1A1
			Total 14

(c)	**Alternative:** Using factor formulae Let ends of the chord be $(2\cos\alpha, \sin\alpha)$ and $(2\cos\beta, \sin\beta)$ **(Must be of this form)**	
	$\left(\cos\alpha + \cos\beta, \frac{\sin\alpha + \sin\beta}{2}\right) = \left(2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right), \sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right)$ M1: Attempt mid-point **and** uses factor formulae A1: Correct mid-point	M1A1
	$m = \frac{\sin\beta - \sin\alpha}{2\cos\beta - 2\cos\alpha} = \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{-4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}\left(= -\frac{1}{2}\cot\left(\frac{\alpha+\beta}{2}\right)\right)$ M1: Attempt gradient **and** uses factor formulae A1: Correct gradient	M1A1
	$y = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)}x$ and $m = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)} \Rightarrow y = -\frac{1}{4m}x$ ddM1: Uses the mid-point and gradient to establish an equation connecting *y*, *m* and *x* **Dependent on both previous method marks** A1: Correct equation	ddM1A1
		(6)

	Special Case: $x^2 + 4y^2 = 4 \Rightarrow 2x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$, *so* $m = -\frac{x}{4y}\left(y = -\frac{1}{4m}x\right)$ **Attempts like these that include further explanation should be sent to review.**	M1A1 **First 2 marks on ePEN**

Alternatives for 5(c)

	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}-2\\-3\\-1\end{pmatrix}+\mu\begin{pmatrix}1\\-2\\-2\end{pmatrix}\Rightarrow 4x-5y+7z=3$	M1: Correctly forms the parametric equation and eliminates the parameters to obtain a cartesian equation A1: Correct cartesian equation	M1A1
	$4x-5y+7z=3\Rightarrow \mathbf{r}.\begin{pmatrix}4\\-5\\7\end{pmatrix}=3$	dM1: Converts their Cartesian equation into the form required. **Dependent on the previous M** A1: Correct equation (oe)	dM1A1

	$a+3b+2c=d$ $-a+c=d$ $2a+b=d$ $\Rightarrow a=\frac{4}{3}d, b=-\frac{5}{3}d, c=\frac{7}{3}d$	M1: Substitutes to obtain 3 equations in *a*, *b*, *c* and *d* and solves to obtain at least one of *a*, *b* or *c* in terms of *d* A1: Correct *a*, *b* and *c* in terms of *d*	M1A1
	$\frac{4}{3}x-\frac{5}{3}y+\frac{7}{3}z=1\Rightarrow \mathbf{r}.\frac{1}{3}\begin{pmatrix}4\\-5\\7\end{pmatrix}=1$	dM1: Uses their cartesian equation correctly to form a vector equation **Dependent on the previous M**	dM1A1